Journal of Mechanical Science and Technology (KSME Int. J.), Vol. 19, No. 9, pp. 1711~1719, 2005 **1711**

Resonance Frequency and Quality Factor Tuning in Electrostatic Actuation of Nanoelectromechanical Systems

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In an electro statically actuated nanoelectromechanical system (NEMS) resonator, it is shown that both the resonance frequency and the resonance quality (Q) factor can be manipulated. How much the frequency and quality factor can be tuned by excitation voltage and resistance on a doubly-clamped beam resonator is addressed. A mathematical model for investigating the tuning effects is presented. All results are shown based on the feasible dimension of the nano resonator and appropriate external driving voltage, yielding up to 20 MHz resonance frequency. Such parameter tuning could prove to be a very convenient scheme to actively control the response of NEMS for a variety of applications.

Key Words : Nano Resonator, Resonance Shift, Quality Factor Shift, Parametric Resonance, NEMS, Electrostatic

1. Introduction

NEMS (nanoelectromechanical systems) are emerging fields of nanotechnology and various consequential fabrications and applications have been reported so far. Most of NEMS devices are with nano scaled dimensions- mostly operated in their resonant modes. In this size regime, NEMS come with extremely high resonance frequencies, diminished active masses, tolerable force constants and high quality (Q) factors of resonance. These attributes collectively to making NEMS suitable for several technological applications such as ultra-fast actuators, sensors, and high frequency signal processing components.

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(Manuscript **Received** December 18, 2004; **Revised** June 15, 2005)

There exist fundamental and technological challenges to NEMS optimization. One of the remaining challenges to developing technologies based upon NEMS is a robust, sensitive and broadband displacement detection method for sub-nanometer displacements. Most displacement sensing techniques used in the domain of microelectromechanical systems (MEMS) are not simply scaleable into the domain of NEMS — necessitating the development of new techniques to realize the full potential of NEMS. One typical NEMS application is a resonator utilizing a resonant frequency by exciting the device with the natural frequency (Carr and Craighead, 1997 ; Sekaric et al., 2002a ; Cleland and Roukes 2002). There are several ways of exciting the resonator ; electrostatic (Carr and Craighead, 1997 ; Sekaric et al., 2002a ; Carr et al., 2000 ; Sekaric et al., 2002b), electromagnetic (Cleland and Roukes 2002 ; Yang et al., 2001) and optical way (Petitgrand et al., 2003 ; Vogel et al., 2003). The electrostatic way has dominated a nano excitation so far due to its relatively easy implementation and feasible force generation to vibrate a clamped beam resonator.

In this work, we introduce a nano resonator consisting of a doubly clamped beam that is functioning in the range of tens of MHz, and the resonant frequency shift due to the applied voltage and external resistance is explored by an analytical approach. Also, the quality factor (inverse of damping coefficient) shift by the external resistance and driving voltage (excitation voltage) is investigated. The main issue in the electrostatic excitation covers how much the excitation voltage affects the stiffness of the resonator, which event ually yields a resonance frequency shift. To investigate the resonance frequency shift in the system with a time-dependent stiffness, a parametric amplification (Rugar and Grutter, 1991 ; Cleland, 2003) has been reported. However, the parametric amplification imposes the assumption that the stiffness and the driving force (applied voltage) should be modulated independently, which means the modulated stiffness and the driving force can be assigned by different frequencies along with phase angle, respectively. In reality, the model of the doubly clamped resonator shows that the modulated stiffness and the driving force are coupled, i.e., they have to keep the same frequency. The quality factor is also coupled with the driving force. In this paper, we analyze how the resonance frequency can be shifted under the applied voltage, and how the quality factor depends on both the applied voltage and the resistance inside the resonator. In order to prove the validity of the analysis several simulations have been done.

2. Analysis

Consider a doubly-clamped nano resonator fabricated by author's team which is completed by sequential nano fabrication processes, and those details on the fabrication processes are explained (Kouh et al., 2004). Utilizing an elastic continuous beam analysis, construct a mechanical vibration model of a double clamped nano beam (Fig. 1). The equation of motion of the beam is

$$\frac{\partial^4 z}{\partial x^4} = -\frac{\rho A}{EI}\frac{\partial^2 z}{\partial t^2} + \frac{F_e(z,\ t)}{EI} \tag{1}$$

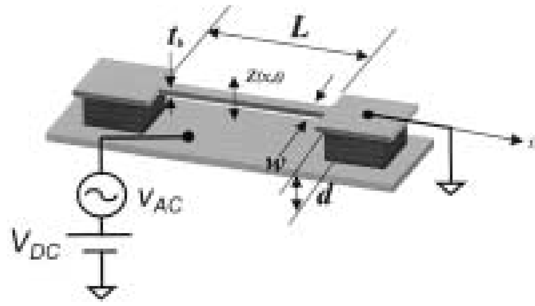

Fig. 1 Transverse motion of double clamped nano beam

where z is the transverse movement of the beam, E is Young's modulus, A is the beam's cross sectional area, ρ is its mass density. $F_e(z,\ t)$ is the applied electrostatic force per unit length in the x-direction, which will be addressed in detail. I is the moment of area of the beam, which is evaluated as $At_b^2/12$ when the beam has a rectangular cross section and thickness of t_b.

By a separation of variables, the solution of the beam equation is written :

$$z(x,\ t) = e^{-i\omega_n t}(\alpha_1 e^{-\lambda_n x} + \alpha_2 e^{\lambda_n x} + \alpha_3 e^{-i\lambda_n x} + \alpha_4 e^{i\lambda_n x}) \tag{2}$$

where λ_n is the wave number, being determined from the boundary conditions, and n is the index representing a particular mode of vibration. ω_n is the angular velocity at mode n. For a clamped beam at both ends, the frequencies of normal modes are obtained by (Yang et al., 2001)

$$f_n = C_n \frac{t}{L^2}\sqrt{\frac{E}{\rho}} \tag{3}$$

Each coefficient up to the third mode is given as $C_1 = 1.03$, $C_2 = 2.83$, and $C_3 = 5.55$, respectively.

Next, considering the infinitesimal movement and uniformity in the longitudinal direction, the beam can be modeled as a lumped system :

$$m\ddot{z} + \frac{m\omega_0}{Q}\dot{z} + m\omega_0^2 z = -F_e \tag{4}$$

where m is the beam mass, Q is the quality factor which is inversely proportional to damping coefficient, and ω_0 is the beam natural frequency. The plates consisting of the resonator become electrically charged when a voltage is applied to the plates, yielding the capacitance in the charged plates, which is given as

$$C=\frac{\varepsilon_r\varepsilon_0 wL}{d-z} \tag{5}$$

where ε_0 is the permittivity in free space and it usually takes $8.85*10^{-12}$ pF/m. ε_r is the relative permittivity (for vacuum=1), and d is the initial gap. L is the beam length, w is the beam width. The electrostatic force exerted on a doubly clamped beam can be driven from differentiating the potential energy by z, and it is expressed as

$$F_e=-\frac{1}{2}\frac{\varepsilon_r\varepsilon_0 wL}{(d-z)^2}V^2 \tag{6}$$

where V is the applied (excited) voltage to the two plates. This force is a primary force to generate a resonance, which allows us to measure an appropriate displacement in a nano scale.

In fabricating a nano beam coated with a metal, there could be an electric passage between two plates even if a complete fabrication has been made. When a doped Si substrate is utilized in a fabrication, it usually generates internal resistance (R). An AC voltage applied to the two plates as well as a capacitance change due to a plate vibration cause a current through the plates. Now, an equivalent to the nano resonator is shown in Fig. 2.

Let q be the electric charge between two plates, yielding $q=CV_c$ by the capacitance C and the capacitor voltage V_c. Here, the capacitor voltage is very close to the excitation voltage. This is shown by the following dynamic analysis.

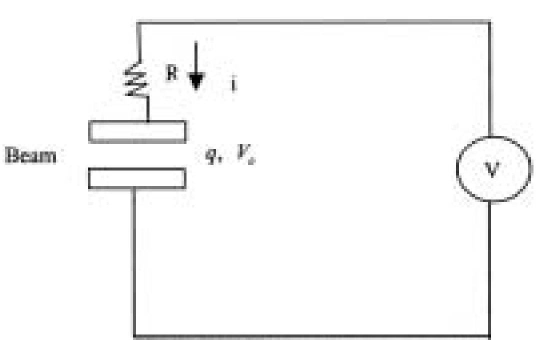

Fig. 2 Equivalent circuit for a nano resonator

$$V=V_c+iR=\frac{q}{C}+iR \tag{7}$$

The current on this circuit is determined as

$$i=\frac{dq}{dt}=\frac{d}{dt}(CV_c)=\frac{d}{dt}(CV)-R\frac{di}{dt} \tag{8}$$

Therefore, the current has the first order differential equation

$$R\frac{di}{dt}+i=\frac{d}{dt}(CV) \tag{9}$$

At the steady state, the current approaches $\frac{d}{dt}(CV)$, which implies the current is dominated by the applied voltage V. The current passing the capacitor depends on both the capacitance and the applied voltage. Moreover, the capacitance varies with time, which is different from the general current calculation in a capacitor (C is usually regarded as constant due to a fixed gap between two plates). Through these conditions, the current is calculated as follows.

$$i=\frac{d}{dt}(CV)=\dot{C}V+C\dot{V} \tag{10}$$

Putting the excitation voltage ($V=V_{dc}+V_{ac}\cos\omega t$), where V_{dc}, and V_{ac} are DC and AC voltages applied the top plate of the resonator with the bottom plate being grounded, then the current in Eq. (10) can be seen that

$$\begin{aligned} i&=\dot{C}V+C\dot{V} \\ &=\frac{d}{dt}\left(\frac{\varepsilon_r\varepsilon_0 wL}{d-z}\right)V \\ &\quad+\frac{\varepsilon_r\varepsilon_0 wL}{d-z}\frac{d}{dt}(V_{dc}+V_{ac}\sin\omega t) \end{aligned} \tag{11}$$

Let $\varepsilon=\varepsilon_r\varepsilon_0 wL$, then the current is simplified as

$$i=\frac{\varepsilon}{(d-z)^2}V\dot{z}-\frac{\varepsilon\omega V_{ac}}{d-z}\cos\omega t \tag{12}$$

Here, we see that the current has a dependency on $\dot{z}$, which has a key to tuning the quality factor along with an excitation voltage.

Next, the electrostatic force (F_e) exerted on the upper plate is computed as follow.

$$\begin{aligned} F_e &= \frac{1}{2}\frac{\varepsilon_r\varepsilon_0 wL}{(d-z)^2}V_e^2 \\ &= \frac{1}{2}\frac{\varepsilon_r\varepsilon_0 wL}{(d-z)^2}(V-iR)^2 \\ &= \frac{1}{2}\frac{\varepsilon_r\varepsilon_0 wL}{(d-z)^2}(V^2-2iRV-i^2R^2) \end{aligned} \tag{13}$$

In a nano scale resonator, $V_R = iR \ll V$, so the third term in Eq. (13) can be negligible. Substituting Eq. (12) into Eq. (13), the electrostatic force can be approximately written as

$$\begin{aligned} F_e &\cong -\frac{1}{2}\frac{\varepsilon}{(d-z)^2}(V^2-2iRV) \\ &= -\frac{1}{2}\frac{\varepsilon}{(d-z)^2}\left(V^2-2RV^2\frac{\varepsilon}{(d-z)^2}\dot{z} \right. \\ &\qquad \left. +2RV\frac{\varepsilon\omega V_{ac}}{d-z}\cos(\omega t)\right) \end{aligned} \tag{14}$$

Again, the third term in Eq. (14) is much less rather than the first or second term because it has only $(d-z)$ term in the denominator ($(d-z)$ in a nano resonator is in the range of nm) which implies $\frac{1}{(d-z)} < \frac{1}{(d-z)^2}$ on the condition of nano scale dimension, and relatively small V_{ac} compared to DC voltage (V_{dc}). The relatively small value of V_{ac} is explained as follow. To drive/detect NEMS resonance and other RF applications, we have to use RF function generator and/or network analyzer. However these RF instruments have limitations in providing PF power. (in RF world, everything is respect to 50 ohms. Power$=\frac{V_{ac}^2}{50}$ and dBm is the relative power from 1 mW). For example, the RF signal generator we have has a maximum RF power of 14 dBm and the network analyzer up to 13 dBm, which is corresponding to 1 V. So we can only drive NEMS within the power limit of these instruments. There are several reasons for this power limitation such as heating of the oscillator inside of the instrument. And as we mentioned before, one of the purpose of miniaturization is to reduce the power level we are applying.

Finally, the electrostatic force is expressed as

$$F_e = -\frac{1}{2}\frac{\varepsilon}{(d-z)^2}\left(V^2-2RV^2\frac{\varepsilon}{(d-z)^2}\dot{z}\right) \tag{15}$$

Now, substituting the derived electrostatic force in Eq. (15) for the beam dynamic equation of Eq. (4), it can be expressed as

$$\begin{aligned} m\ddot{z}+\frac{m\omega_0}{Q}\dot{z}+m\omega_0^2 z &= -F_e \\ &= \frac{1}{2}\frac{\varepsilon V^2}{(d-z)^2}-\frac{R\varepsilon^2V^2}{(d-z)^4}\dot{z} \end{aligned} \tag{16}$$

The terms in the right side in Eq. (16) have nonlinearity in terms of z, and $\dot{z}$, but these can be linearized by employing a linearization technique, which is quite meaningful in a nano resonator with respect to its infinitesimal displacement change. Then, the linearized dynamics of the nano resonator can be seen that

$$\begin{aligned} & m\ddot{z}+\frac{m\omega_0}{Q}\dot{z}+m\omega_0^2 z \\ &= \frac{1}{2}\left(\frac{1}{d^2}+\frac{2}{d^3}\right)\varepsilon V^2 - R\varepsilon^2V^2\left(\frac{4}{d^5}z+\frac{1}{d^4}\dot{z}\right) \end{aligned} \tag{17}$$

The resonator has a standard form after arranging terms of z and $\dot{z}$:

$$\begin{aligned} & m\ddot{z}+\left(\frac{mw_0}{Q}+\frac{R\varepsilon^2V^2}{d^4}\right)\dot{z} \\ & +\left(m\omega_0^2-\frac{\varepsilon V^2}{d^3}+\frac{4R\varepsilon^2V^2}{d^5}\right)z \\ & = \frac{1}{2d^2}\varepsilon V^2 \end{aligned} \tag{18}$$

It is implicative that the quality factor and the stiffness are affected by the external excitation voltage and the resistance. Viewing Eq. (18), we define an effective quality factor and resonance frequency which is shifted from the fundamental quality factor (Q) and frequency (ω_0), respectively.

$$\begin{aligned} \frac{1}{Q_{eff}} &= \frac{1}{Q}+\frac{R\varepsilon^2V^2}{m\omega_0 d^4} \\ \omega_{eff}^2 &= \omega_0^2-\frac{\varepsilon V^2}{md^3}+\frac{4R\varepsilon^2V^2}{md^5} \end{aligned} \tag{19}$$

As being inferred from this result, it is shown that the resonance frequency and the quality factor are shifted by both applied voltage and resistance. However, the excitation voltage consists of AC (time-varying) and DC components, hence it makes the direct use of Eq. (19) difficult to calculate the variations of frequency and quality factor. From now on, how effectively the applied voltage consisting AC voltage (V_{ac}) and DC voltage (V_{dc}), contributes to the resonance frequency change is investigated. The applied voltage is given by

$$V^2 = V_{dc}^2 + 2V_{dc}V_{ac}\cos\omega t + \frac{V_{ac}^2(1-\cos 2\omega t)}{2} \tag{20}$$

As stated before, due to the power limitation of a network analyzer (an excitation instrument), the amplitude of V_{ac} needs to be set much lower than DC voltage. Thus V^2 can be approximated as

$$V^2 = V_{dc}^2 + 2V_{dc}V_{ac}\cos\omega t \tag{21}$$

From this voltage representation in Eq. (21), the beam equation of Eq. (18) ends up with

$$\begin{aligned} &m\ddot{z} + \left(\frac{m\omega_0}{Q} + \frac{R\varepsilon^2}{d^4}\left(V_{dc}^2 + 2V_{dc}V_{ac}\cos(\omega t)\right)\right)\dot{z} \\ &+\left(m\omega_0^2 - \left(\frac{\varepsilon}{d^3} - \frac{4R\varepsilon^2}{d^5}\right)\left(V_{dc}^2 + 2V_{dc}V_{ac}\cos(\omega t)\right)\right)z \\ &= \frac{\varepsilon}{2d^2}\left(V_{dc}^2 + 2V_dV_{ac}\cos(\omega t)\right) \\ &= m\ddot{z} + (k_1 + 2k_2\cos\omega t)\dot{z} + (k_3 + 2k_4\cos\omega t)z \\ &= k_5 + 2k_6\cos\omega t \end{aligned} \tag{22}$$

where

$$\begin{aligned} &k_1 = \frac{m\omega_0}{Q} + \frac{R\varepsilon^2}{d^4}V_{dc}^2,\ k_2 = \frac{R\varepsilon^2}{d^4}V_{dc}V_{ac} \\ &k_3 = m\omega_0^2 - \left(\frac{\varepsilon}{d^3} - \frac{4R\varepsilon^2}{d^5}\right)V_{dc}^2 \\ &k_4 = -\left(\frac{\varepsilon}{d^3} - \frac{4R\varepsilon^2}{d^5}\right)V_{dc}V_{ac} \\ &k_5 = \frac{\varepsilon}{2d^2}V_{dc}^2,\ k_6 = \frac{\varepsilon}{2d^2}V_{dc}V_{ac} \end{aligned} \tag{23}$$

Seeing the stiffness and the quality factor, it is imperative to understand those depend on AC, DC voltage, and resistance in the resonator. Taking a glance at Eq. (22), the increment of AC and DC voltage makes the resonance frequency be less than the natural frequency of the resonator (free vibration) even if the correct calculation of the shift remains unclear due to the $\cos(\omega t)$ term. Moreover, the resistance seems to affect the quality factor to be less than that of the system without resistance.

Now, we try to analyze quantitatively how much the voltage and resistance have an influence on the resonance frequency and the quality factor by solving the system. Let the solution of the system be

$$z(t) = Ae^{-i\omega t} + Be^{i\omega t} + D \tag{24}$$

where A, B, and D are complex values depending on the excitation frequency ω. Substituting Eq. (24) into Eq. (22), performing algebraic calculations, and replacing $\cos\omega t = \frac{e^{i\omega t} + e^{-i\omega t}}{2}$, the equation comprising of $e^{i\omega t}$, $e^{-i\omega t}$, and a constant (e^0) can be rearranged. Retaining the $e^{i\omega t}$ term it is found to be

$$-m\omega^2 A - ik_1\omega A + k_3 A + k_4 D = k_6 \tag{25}$$

For the $e^{i\omega t}$ term, it is found to be

$$-m\omega^2 B + ik_1\omega B + k_3 B + k_4 D = k_6 \tag{26}$$

For a constant (e^0) term, it is expressed as

$$k_3 D + (k_4 - ik_2\omega)A + (ik_2\omega + k_4)B = K_5 \tag{27}$$

Then the above three equations Eqs. (25-27) are represented as a matrix form :

$$\begin{bmatrix} L_1(\omega) & 0 & L_4 \\ 0 & L_2(\omega) & L_4 \\ L_5(\omega) & L_6(\omega) & L_3 \end{bmatrix}\begin{bmatrix} A \\ B \\ D \end{bmatrix} = \begin{bmatrix} k_6 \\ k_6 \\ k_o \end{bmatrix} \tag{28}$$

where all elements are given as

$$\begin{aligned} L_1(\omega) &= -m\omega^2 - ik_1\omega + k_3 \\ L_2(\omega) &= -m\omega^2 + ik_1\omega + k_3 \\ L_3(\omega) &= k_3 \\ L_4 &= k_4 \\ L_5(\omega) &= k_4 - ik_2\omega \\ L_6(\omega) &= k_4 + ik_2\omega \end{aligned} \tag{29}$$

The coefficients A, B and D are computed as follows from Eq. (28)

$$A=\frac{L_2(L_3k_6-L_4k_5)}{L_1(L_2L_3-L_4L_6)-L_4L_2L_5}$$
$$B=\frac{L_1(L_3k_6-L_4k_5)}{L_1(L_2L_3-L_4L_6)-L_4L_2L_5} \quad (30)$$
$$D=\frac{-k_6(L_2L_5+L_1L_6)+L_1L_2k_5}{L_1(L_2L_3-L_4L_6)-L_4L_2L_5}$$

Here, the common denominator of A, B and D, which is a determinant of the matrix in Eq. (28), yields the simplified form

$$\begin{aligned}&L_1(L_2L_3-L_4L_6)-L_4L_2L_5\\&=k_3[(-m\omega^2+k_3)+k_1^2\omega^2]\\&\quad-2[k_4(-m\omega^2+k_3)+k_4k_1k_2\omega^2]\\&=:k_3G(\omega)^2-2k_4G(\omega)+(k_3k_1^2-2k_4k_1k_2)\,\omega^2\end{aligned} \quad (31)$$

where $G(\omega)=-m\omega^2+k_3$. It is seen that $G(\omega)=0$ in Eq. (31) makes the determinant be close to the minimum. The true minimum value is calculated later. The corresponding frequency achieving the approximate minimum of the determinant almost gives rise to the resonance at the values of A, B, and D, almost maximizing the displacement of beam $z(t)$. By the condition of $G(\omega)=0$, and the stiffness $k=m\omega_0^2$, the shifted resonance frequency (ω_c) is determined as

$$\begin{aligned}\omega_c^2&=\frac{k_3}{m}=\omega_0^2-\frac{1}{m}\left(\frac{\varepsilon}{d^3}-\frac{4R\varepsilon^2}{d^5}\right)V_{dc}^2\\&=\omega_0^2\left(1-\left(\frac{\varepsilon}{kd^2}-\frac{4R\varepsilon^2}{kd^5}\right)V_{dc}^2\right)\end{aligned} \quad (32)$$

More accurately, the minimum value (ω_m) considering all terms in Eq. (31) can be determined as

$$\omega_m^2=\frac{k_3}{m}-\frac{k_1^2}{2m^2}-\frac{k_4}{k_3m}+\frac{k_1k_2}{k_3m^2} \quad (33)$$

Comparing with two frequencies (ω_c and ω_m), there is not much discrepancy between them since the first term in Eq. (33) is dominant and the others are extremely small in a nano scale resonator, eventually closely equal to Eq. (32). As a consequence, the resonance frequency in a nano resonator adopts the result of Eq. (32) for a brevity of calculation, which is shifted by $\left(\frac{\varepsilon}{md^3}-\frac{4R\varepsilon^2}{md^5}\right)V_{dc}^2$ from the resonance frequency of the original beam (ω_0). In other words, the increment of DC voltage makes the resonance be less than the natural frequency of the nano beam, but the resistance affects it reversely.

Figures 3∼5 show the time responses under different excitation frequencies, ensuring that only resonance appears at resonance frequency (ω_c) and other off-resonance frequency excitations show the displacement damps out as time goes on. The nano resonator taken for consideration here has 400 nm thickness, 10 μm length, and 1 μm width. The magnitude of the driving AC voltage

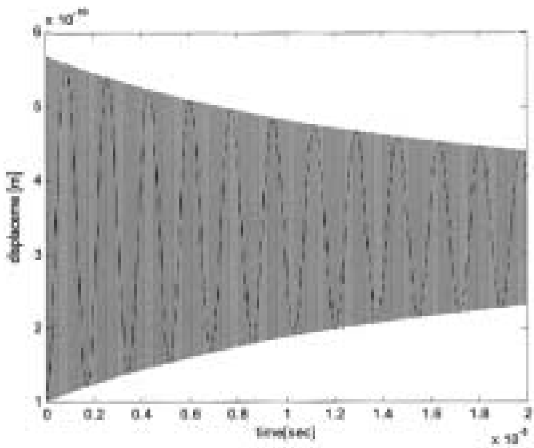

Fig. 3 Time response of nano beam with an excitation frequency $\omega=5\omega_c$

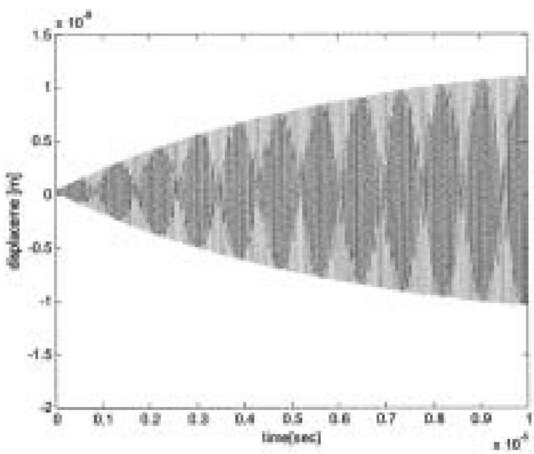

Fig. 4 Time response of nano beam with an excitation frequency $\omega=\omega_c$

is given by 0.5 V, which is about 7 dBm in power, being within the power limitation in a network analyzer (13 dBm). DC voltage varies from 1 V to 14 V, mainly attributing to a static deflection of the beam, which corresponds with the condition that the AC voltage is much smaller than the DC voltage. The other mechanical properties are selected as $E=107*10^9\,\mathrm{Nm}^2$, $\rho=2330\,\mathrm{kg/m^3}$, $t_b=200\,\mathrm{nm}$ $d=400\,\mathrm{nm}$ $L=8\,\mu\mathrm{m}$ $w=1\,\mu\mathrm{m}$, $\varepsilon_r=1$, $\varepsilon_0=8.85\,\mathrm{pF/m}$. It is crucial that the parametric resonance $\left(2\omega_0, \frac{2\omega_0}{3}, \frac{\omega_0}{2}, \cdots\right)$ does not appear at the electrostatic resonator, resulting in a forced vibration problem. These results show difference from the parametric resonance case (Carr et al., 2000 ; Cleland, 2003) because the effective stiffness (original stiffness+applied sinusoidal voltage) is surly coupled with the driving force. The "coupled" here means that the time-variant stiffness and the driving force can not be modulated independently, being contrary to the assumption adopted in (Rugar and Grutter, 1991 ; Cleland, 2003).

Fig. 6 shows the frequency response of the coefficient A in Eq. (30). Viewing the result, the resonance frequency is decreased as V_{dc} increases for a fixed resistance, which is already proved analytically before. Fig. 7 shows the resonance frequency change according to the V_{dc} variations. As expected, the frequency decreases as V_{dc} increases. However, the resistance dose not affect the resonance frequency noticeably, which is proved by calculating each term in Eq. (32). For given $\omega_0=1.34*10^8$ rad/sec and $R=105$ ohms, each term in Eq. (32) has $m\omega_0^2=69.8$, $\frac{\varepsilon V_{dc}^2}{d^3}=8.1$, $\frac{4R\varepsilon^2 V_{dc}^2}{d^5}=3.2*10^{-3}$, respectively. The effect on the stiffness by excitation voltage, as shown at the third term in Eq. (32), is extremely smaller than the first or the second term. Therefore, the third term can be negligible, and the effect on the resonance frequency by resistance is verified in Fig. 8. The experimental results on resonance frequency shift are shown in (Pourkamali et al., 2003).

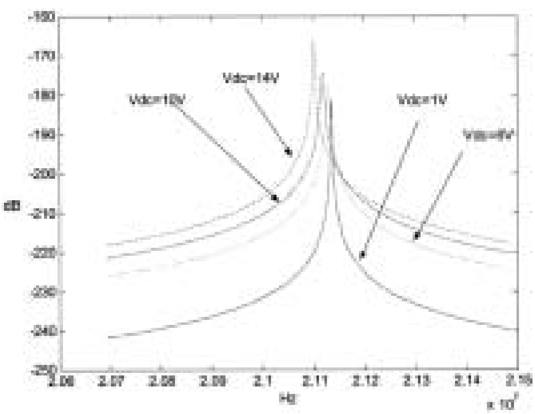

Fig. 6 Resonance frequency shift due to the variation of V_{dc}

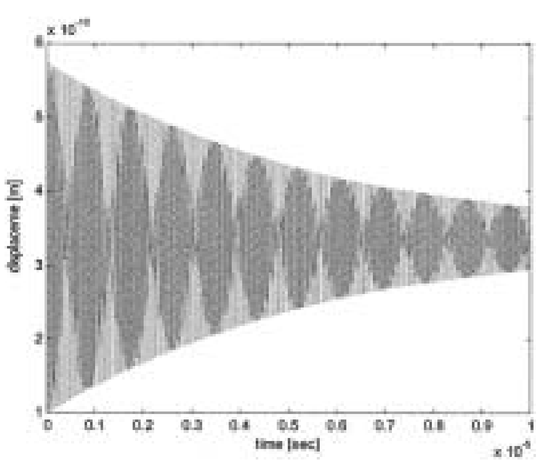

Fig. 5 Time response of nano beam with an excitation frequency $\omega=2\omega_c$ (one of the parametric resonances)

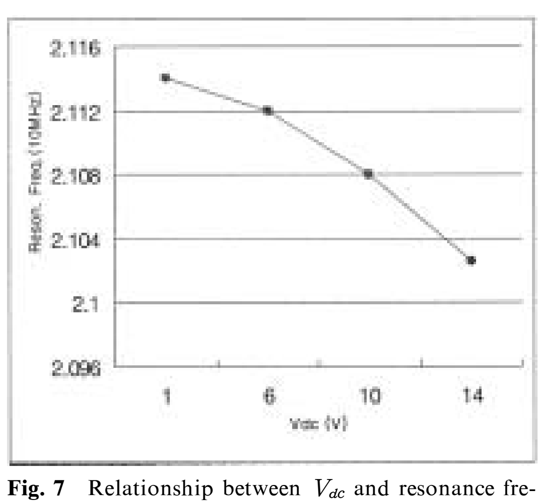

Fig. 7 Relationship between V_{dc} and resonance frequency

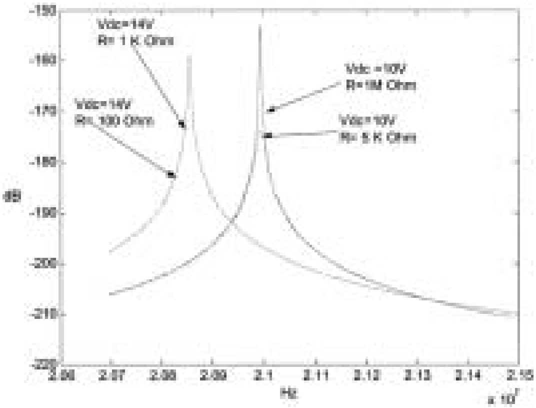

Fig. 8 Quality factor change due to the resistance variation (gap d=200 nm)

The quality factor is also affected by DC voltage and resistance, and new quality factor is determined from k_1 in Eq. (23) as

$$\frac{1}{Q_c}=\frac{1}{Q}+\frac{R\varepsilon^2 V_{dc}^2}{m\omega_0 d^4} \tag{34}$$

Therefore, the DC voltage and the resistance decrease the quality factor. Fig. 8 shows the quality factor change as the resistance varies, from which the resistance is considered as a dominant factor to induce the change. If the driving frequency is such that $\frac{2\omega_0}{\omega_0-\omega}=\frac{1}{Q_c}$, which corresponds to the amplitude of $\frac{1}{\sqrt{2}}$ times of the maximum resonance magnitude. This allows the measurement of the quality factor Q_c by direct measurement of the resonance peak from the resonance frequency diagram. The quality factor in Fig. 8 is around 900∼5000, and the quality factors for each case are shown in Table 1. To show more distinguishable change, the gap is set by 200 nm from the original 400 nm. On the other hand, V_{dc} is not effective in the quality factor. As for a beam with larger width and length, and smaller gap, and with high resistance and high voltage source, it is adequate to claim that the quality factor can be decreased considerably viewing Eq. (34). Again, if R=0, then there is no change on the quality factor even if voltage is applied.

Finally, if an internal resistance (a sort of extrinsic dissipation under incomplete insulation between the substrate and the beam) exists inherently in a nano resonator it gives rise to a change on a quality factor along with an excitation voltage. On the other hand, the resonance frequency is mainly affected by V_{dc} rather than the resistance.

Table 1 Quality factors for various resistance and DC voltage

Resistance	V_{dc}	Q_c (quality factor)
1 M Ohm	10V	4620
5 K Ohm	10V	924
1 K Ohm	14V	2294
100 Ohm	14V	2294

3. Conclusions

A theoretical study on a nano resonator is addressed, and out of plane fundamental resonance of silicon NEMS with dimensions as small as 10 μm × 400 nm × 1000 nm is considered. It is shown that the resonant frequency is about 21 MHz and quality factor lies in the range of $900 < Q < 5000$.

The nano resonator considered in this article excludes a parametric resonance requiring an independent modulation between time-varied stiffness and driving force (applied voltage), resulting in a resonance frequency and a quality factor shifts under applied DC voltage, and resistance on the two plates. The validity of the analysis on the tuning of the resonance frequency and quality factor are verified through simulations. The applied DC voltage and the inherent resistance on the resonator should be precisely selected in designing a nano resonator by considering the shift on the resonance frequency and the quality factor. As long as the precise tuning of the resonator is carried out, the more accurate measurement is accomplished in the applications of a nano resonator.

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